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A METHOD FOR CALCULATING EXACT GEODETIC LATITUDE AND ALTITUDE

BY ISAAC SOFAIR
STRATEGIC SYSTEMS DEPARTMENT

APRIL 1985
REVISED MARCH 1993

NAVAL SURFACE WARFARE CENTER
DAHLGREN DIVISION
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FOREWORD

The work described in this report was performed in the Fire Control Formulation Branch (K41), SLBM Research and Analysis Division, Strategic Systems Department, and was authorized under Strategic Systems Program Office Task Assignment 36401. This work was necessitated by the need to formulate an exact method of computing geodetic latitude and altitude.

This report has been reviewed and approved by Davis Owen; Johnny Boyles; Robert Gates, Head, Fire Control Formulation Branch; and Sheila Young, Head, SLBM Research and Analysis Division.

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INTRODUCTION

Following is a simple and efficient model for calculating the exact geodetic latitude and altitude of an arbitrary point in space, given the coordinates of that point. The model assumes the earth to be an oblate spheroid; i.e., an ellipsoid of revolution whose semimajor axis (a) is the radius of the circle described by the equatorial plane, and whose semiminor axis (b) is a line joining its center and one of its poles. The point in question can be either inside or outside the ellipsoid, but not too close to the center of the ellipsoid. The solution of the problem and its constraints will now be expounded.

FORMULATION

Choose an earth-fixed Cartesian coordinate system whose origin coincides with the center of the ellipsoid. The unit vectors $\hat{i}$, $\hat{j}$, $\hat{k}$ coincide with the (x, y, z) axes, respectively. The +z axis points in the direction of the North Pole. The +x axis is the line of intersection of the equatorial plane and the plane of zero longitude. The +y axis completes a right-handed coordinate system.

An equation of the ellipsoid in this frame is

$$\frac{R^2}{a^2} + \frac{z^2}{b^2} = 1, \quad \text{where} \quad R = \sqrt{x^2 + y^2}\ . \tag{1}$$

Let $P(x_0, y_0, z_0)$ be the coordinates of the given point. It is desired to find the points $P(x, y, z)$ at which the normals from P to the surface of the ellipsoid cut the ellipsoid.

The slope of a normal to the ellipsoid at any point on its surface is given by

$$-\frac{1}{\frac{dz}{dR}} = \frac{a^2 z}{b^2 R}\ , \quad \text{where} \quad \frac{R^2}{a^2} + \frac{z^2}{b^2} = 1\ . \tag{2}$$

Therefore the slope of a normal from P is

$$\frac{z_0 - z}{R_0 - R} = \frac{a^2 z}{b^2 R}\ , \quad \text{where} \quad R_0 = \sqrt{x_0^2 + y_0^2}\ ,$$

i.e.,

$$(R_0 - R)a^2 z = (z_0 - z)b^2 R ,$$

or

$$R_0 a^2 z = R\{a^2 z + b^2(z_0 - z)\} = R\{(a^2 - b^2)z + b^2 z_0\} .$$

Squaring both sides and expressing R in terms of z,

$$a^2 b^2 R_0^2 z^2 = (b^2 - z^2)\{(a^2 - b^2)z + b^2 z_0\}^2 .$$

Writing the above equation in descending powers of z, we obtain the following biquadratic in z:

$$(a^2 - b^2)^2 z^4 + 2b^2(a^2 - b^2)z_0 z^3 + b^2\{a^2 R_0^2 + b^2 z_0^2 - (a^2 - b^2)^2\}z^2 -$$

$$2b^4(a^2 - b^2)z_0 z - b^6 z_0^2 = 0.$$

Now z is either positive or negative and therefore is not an appropriate variable for the solution. However, this difficulty is easily circumvented by introducing a new dimensionless variable $k = \dfrac{z}{z_0}$. Using the constraint that z always has the same sign as z_0, we see that k is always positive. Putting $z = z_0 k$ in the biquadratic and writing the resulting equation such that the coefficient of k^4 is unity, we finally obtain

$$k^4 + 2pk^3 + qk^2 + 2rk + s = 0 \qquad (*)$$

where

$$p = \frac{b^2}{a^2 - b^2} \qquad (3)$$

$$q = \frac{b^2\{a^2 R_0^2 + b^2 z_0^2 - (a^2 - b^2)^2\}}{(a^2 - b^2)^2 z_0^2} \qquad (4)$$

$$r = -\frac{b^4}{(a^2 - b^2)z_0^2} \qquad (5)$$

$$s = -\frac{b^6}{(a^2 - b^2)^2 z_0^2} \qquad (6)$$

Now we will employ two of three powerful theorems in the Theory of Equations to expose the nature of the roots of (*). The third theorem will be needed later on. The proofs of these theorems are given in Reference 1.

I. An equation $f(x) = 0$ cannot have more positive roots than there are changes of sign in $f(x)$, and cannot have more negative roots than there are changes of sign in $f(-x)$.

II. Every equation which is of an even degree and has its last term negative has at least two real roots, one positive and one negative.

III. Every equation of an odd degree has at least one real root whose sign is opposite to that of its last term.

By observing (*) and (3), (4), (5), (6), we deduce at once the following conclusions:

(i) Since the last term of (*), namely s, is negative, there are at least two real roots of opposite sign (using II).

(ii) Since there is only one change of sign in (*), irrespective of whether q is positive or negative, there is at most one positive root (using I).

From (i) and (ii) we see immediately that (*) has exactly one positive root. Since k is constrained to be positive, this positive root is the one we seek.

SOLUTION OF BIQUADRATIC

The solution of (*) is effected by a standard method known as Ferrari's method which will now be enunciated.

In the equation

$$k^4 + 2pk^3 + qk^2 + 2rk + s = 0$$

add to each side $(ck + d)^2$, the quantities c and d being determined so as to make the left-hand side a perfect square; then

$$k^4 + 2pk^3 + (q + c^2)k^2 + 2(r + cd)k + s + d^2 = (ck + d)^2.$$

Suppose that the left side of the equation is equal to $(k^2 + pk + t)^2$, then by comparing the coefficients, we have

$$p^2 + 2t = q + c^2, \quad pt = r + cd, \quad t^2 = s + d^2. \tag{7}$$

By eliminating c and d from these equations, we obtain

$$(pt - r)^2 = (2t + p^2 - q)(t^2 - s),$$

$$\text{or } 2t^3 - qt^2 + 2(pr - s)t - p^2 s + qs - r^2 = 0. \tag{8}$$

From (3), (4), (5), and (6), we see that

$$pr - s = 0, \ -p^2 s + qs - r^2 = -\frac{a^2 b^8 R_0^2}{(a^2 - b^2)^4 z_0^4} .$$

Substituting into (8), we obtain the following cubic in t:

$$2t^3 - qt^2 - \frac{a^2 b^8 R_0^2}{(a^2 - b^2)^4 z_0^4} = 0 . \tag{8'}$$

Applying Theorem III to (8'), we see that (8') has at least one positive root. Now applying Theorem I to (8'), we see that it has at most one positive root, irrespective of whether q is positive or negative. Hence (8') has exactly one positive root.

The solution of a cubic is accomplished by a standard method known as Cardan's Solution.

First eliminate the t^2 term in (8') by making the substitution

$$t = t' + \frac{q}{6} ,$$

i.e.,

$$2(t' + \frac{q}{6})^3 - q(t' + \frac{q}{6})^2 - \frac{a^2 b^8 R_0^2}{(a^2 - b^2)^4 z_0^4} = 0$$

or

$$t'^3 - \frac{q^2}{12} t' - \frac{q^3}{108} - \frac{a^2 b^8 R_0^2}{2(a^2 - b^2)^4 z_0^4} = 0 .$$

Let

$$f = -\frac{q^2}{12} , \ h = -\frac{q^3}{108} - \frac{a^2 b^8 R_0^2}{2(a^2 - b^2)^4 z_0^4}$$

so that the above equation can be written in the form

$$t'^3 + ft' + h = 0 . \tag{9}$$

To solve (9), let $t' = u + v$; then

$$t'^3 = (u + v)^3 = u^3 + v^3 + 3uv\,(u + v) = u^3 + v^3 + 3uvt'$$

and (9) becomes

$$u^3 + v^3 + (3uv + f)t' + h = 0.$$

At present, u and v are any two quantities subject to the condition that their sum is equal to one of the roots of (9); if we further suppose that they satisfy the equation $3uv + f = 0$, they are completely determinate. We thus obtain

$$u^3 + v^3 = -h, \quad u^3v^3 = -\frac{f^3}{27} .$$

Hence, u^3, v^3 are the roots of the quadratic

$$\alpha^2 + h\alpha - \frac{f^3}{27} = 0 .$$

Thus,

$$\alpha = -\frac{h}{2} \pm \sqrt{\frac{h^2}{4} + \frac{f^3}{27}} .$$

Putting $u^3 = -\frac{h}{2} + \sqrt{\frac{h^2}{4} + \frac{f^3}{27}}$, $v^3 = -\frac{h}{2} - \sqrt{\frac{h^2}{4} + \frac{f^3}{27}}$, we obtain t' from the relation $t' = u + v$.

Hence, $$t' = \left(-\frac{h}{2} + \sqrt{\frac{h^2}{4} + \frac{f^3}{27}}\right)^{1/3} + \left(-\frac{h}{2} - \sqrt{\frac{h^2}{4} + \frac{f^3}{27}}\right)^{1/3}. \tag{10}$$

The solution (10) is valid provided that $\frac{h^2}{4} + \frac{f^3}{27} \geq 0$. It will now be shown that this constraint is valid for all points of interest.

$$\frac{h^2}{4} + \frac{f^3}{27} = \frac{1}{4}\left\{-\frac{q^3}{108} - \frac{a^2 b^8 R_0^2}{2(a^2 - b^2)^4 z_0^4}\right\}^2 - \frac{q^6}{46656}$$

$$= \frac{a^2 b^8 R_0^2 q^3}{432(a^2 - b^2)^4 z_0^4} + \frac{a^4 b^{16} R_0^4}{16(a^2 - b^2)^8 z_0^8}$$

$$= \frac{a^2 b^{14} R_0^2 \{a^2 R_0^2 + b^2 z_0^2 - (a^2 - b^2)^2\}^3}{432(a^2 - b^2)^{10} z_0^{10}} + \frac{a^4 b^{16} R_0^4}{16(a^2 - b^2)^8 z_0^8}, \text{ using (4)}$$

$$= \frac{a^2 b^{14} R_0^2}{432(a^2 - b^2)^{10} z_0^{10}} [\{a^2 R_0^2 + b^2 z_0^2 - (a^2 - b^2)^2\}^3 +$$

$$27a^2 b^2 (a^2 - b^2)^2 R_0^2 z_0^2] .$$

Thus, the required condition is

$$\{a^2 R_0^2 + b^2 z_0^2 - (a^2 - b^2)^2\}^3 + 27a^2 b^2 (a^2 - b^2)^2 R_0^2 z_0^2 \geq 0 \ . \qquad (11)$$

By inspection and also by trial, it is seen that the above constraint is valid at all points of interest. As a matter of fact, the only points where it does not hold are those that are relatively close to the origin, i.e., the center of the ellipsoid; and these points are of no practical interest.

Now

$$h = -\frac{q^3}{108} - \frac{a^2 b^8 R_0^2}{2(a^2 - b^2)^4 z_0^4}$$

$$= -\frac{b^6\{a^2 R_0^2 + b^2 z_0^2 - (a^2 - b^2)^2\}^3}{108(a^2 - b^2)^6 z_0^6} - \frac{a^2 b^8 R_0^2}{2(a^2 - b^2)^4 z_0^4}$$

$$= -\frac{b^6}{108(a^2 - b^2)^6 z_0^6} [\{a^2 R_0^2 + b^2 z_0^2 - (a^2 - b^2)^2\}^3 + 54a^2 b^2 (a^2 - b^2)^2 R_0^2 z_0^2].$$

If the constraint (11) holds, then the expression in the brackets above is positive. Hence, h is negative. Applying Theorem III to (9), we conclude that (9) has at least one positive root. Now applying Theorem I to (9) and using the fact that both f and h are negative, we conclude that (9) has at most one positive root. Hence, (9) has exactly one positive root. It will now be shown that this positive root is given by (10).

Let $g = \frac{h^2}{4} + \frac{f^3}{27}$, then

$$u = \left(\sqrt{g} - \frac{h}{2}\right)^{1/3}, \quad v = \left(-\sqrt{g} - \frac{h}{2}\right)^{1/3} .$$

Now $g - \frac{h^2}{4} = \frac{f^3}{27} < 0$, since $f = -\frac{q^2}{12} < 0$.

Therefore, $(\sqrt{g} - \frac{h}{2})(\sqrt{g} + \frac{h}{2}) < 0$, which implies that $\sqrt{g} - \frac{h}{2}$ and $\sqrt{g} + \frac{h}{2}$ are of opposite sign; however, since $h < 0$, $\sqrt{g} - \frac{h}{2} > \sqrt{g} + \frac{h}{2}$. Hence, $\sqrt{g} - \frac{h}{2} > 0 > \sqrt{g} + \frac{h}{2}$; i.e., $u > v > 0$. Thus, $t' = u + v > 0$.

Proceeding with our derivation, we have

$$t = t' + \frac{q}{6} > 0 \ .$$

Solving for c and d from the system of equations (7), we have

$$c = \sqrt{p^2 - q + 2t} \ , \ d = \sqrt{t^2 - s} \ .$$

Now $(k^2 + pk + t)^2 = (ck + d)^2$,

or $k^2 + pk + t = \pm (ck + d)$

from which we obtain the two quadratics in k:

$$k^2 + (p - c)k + t - d = 0 \tag{12}$$

$$k^2 + (p + c)k + t + d = 0 \tag{13}$$

From (12) and (13), we obtain the following four roots of (*):

$$k_1 = \frac{-(p - c) + \sqrt{(p - c)^2 - 4(t - d)}}{2} \ ,$$

$$k_2 = \frac{-(p - c) - \sqrt{(p - c)^2 - 4(t - d)}}{2} \ ,$$

$$k_3 = \frac{-(p + c) + \sqrt{(p + c)^2 - 4(t + d)}}{2} \ ,$$

$$k_4 = \frac{-(p + c) - \sqrt{(p + c)^2 - 4(t + d)}}{2} \ .$$

DERIVATION OF GEODETIC ALTITUDE AND LATITUDE

It has already been shown that (*) has exactly one positive root. Since $t^2 - d^2 = s < 0$, $(t + d)(t - d) < 0$, which implies that $t + d$ and $t - d$ are of opposite sign; but $d > 0$; thus $t + d > 0 > t - d$. Now $t - d$ is the last term of (12) and has just been shown to be negative. Hence, by applying Theorem II to (12), we see immediately that (12) must contain the desired positive root. Inspecting the first two roots above, it is clear that k_1 is the required positive root.

The rest of the solution follows trivially. Let $k = k_1$, then

$$z = z_0 k \ ,$$

$$R = a\sqrt{1 - \frac{z^2}{b^2}} \quad , \text{ and}$$

$$\left.\begin{aligned} x &= x_0 \frac{R}{R_0} \quad , \\ y &= y_0 \frac{R}{R_0} \quad , \\ \lambda &= \tan^{-1} \frac{y}{x} \end{aligned}\right\} \quad \text{where } R_0 \neq 0 \quad .$$

(Here λ is the longitude at (x, y)).

The above solution is not applicable to the case $z_0 = 0$ since two of the basic parameters of the solution, q and s, involve a division by z_0. Also, the solution is not applicable to the case $R_0 = 0$. Hence these two cases have to be treated separately; nevertheless, this poses no difficulty since the results are already known then.

The geodetic altitude is given by

$$H = \text{sign}\left(\frac{R_0^2}{a^2} + \frac{z_0^2}{b^2} - 1\right)\sqrt{(x_0 - x)^2 + (y_0 - y)^2 + (z_0 - z)^2}$$

$$= \text{sign}\left(\frac{R_0^2}{a^2} + \frac{z_0^2}{b^2} - 1\right)\sqrt{(R_0 - R)^2 + (z_0 - z)^2}$$

In order to find the geodetic latitude and the unit normal vector at (x, y, z), it is desirable to introduce a geometric term, Ne, which is never zero. Ne is defined to be the distance along the ellipsoidal normal from the surface of the ellipsoid to the z-axis (Reference 2). (See Figure 1.)

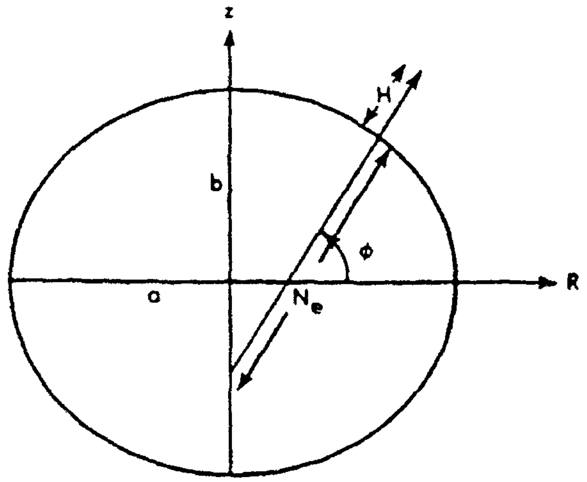

FIGURE 1. ELLIPSOIDAL NORMAL

From Figure 1, we have

$$\cos \phi = \frac{R}{Ne} \quad .$$

Also,

$$\tan \phi = \frac{a^2 z}{b^2 R} \quad , \text{ from (2).}$$

Hence,

$$Ne = R \sec \phi = R \sqrt{1 + \tan^2 \phi} = R \sqrt{1 + \frac{a^4 z^2}{b^4 R^2}} = \frac{\sqrt{a^4 z^2 + b^4 R^2}}{b^2} \quad .$$

$$\sin \phi = \cos \phi \tan \phi = \frac{a^2 z}{b^2 Ne} \quad .$$

Thus, the components of the unit normal vector at (x, y, z) are

$$H_1 = \frac{x}{Ne} \; , \; H_2 = \frac{y}{Ne} \; , \; H_3 = \left(\frac{a}{b}\right)^2 \frac{z}{Ne} \quad .$$

The geodetic latitude ϕ, is

$$\phi = \sin^{-1} H_3 \quad .$$

This completes our solution.

CONCLUSION

In contrast to other methods currently in use, this method theoretically computes the exact values of the geodetic latitude and altitude, assuming the earth to be a perfect ellipsoid of revolution. The method was programmed and run for many given values of x_0, y_0, z_0. It gave results which are consistent with those of other methods, including Brookshire's approximation (Reference 3) and an iterative technique (Reference 4). Roundoff errors are negligible. For all practical purposes, the results are excellent and the method is definitely recommended as an alternative approach to problems involving geodetic latitude.

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APPENDIX

THE INVERSE PROBLEM

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An arbitrary point in space is defined by λ, ϕ, and H. Solve for (H_1, H_2, H_3), (x, y, z), and (x_0, y_0, z_0).

This problem is relatively easy to solve. From page 9, we have

$$R = Ne \cos \phi, \quad z = \frac{b^2}{a^2} Ne \sin \phi.$$

Using the relation $\frac{R^2}{a^2} + \frac{z^2}{b^2} = 1$, we obtain

$$\frac{Ne^2}{a^2} \left(\cos^2 \phi + \frac{b^2}{a^2} \sin^2 \phi\right) = 1;$$

or

$$Ne = \frac{a}{\sqrt{\cos^2 \phi + \frac{b^2}{a^2} \sin^2 \phi}} = \frac{a}{\sqrt{1 - (1 - \frac{b^2}{a^2}) \sin^2 \phi}} .$$

Also, from page 8, we have

$$\lambda = \tan^{-1} \frac{y}{x} = \sin^{-1} \frac{y}{R} = \cos^{-1} \frac{x}{R} .$$

Therefore,

$$x = R \cos \lambda = Ne \cos \phi \cos \lambda,$$

$$y = R \sin \lambda = Ne \cos \phi \sin \lambda.$$

Hence,

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} Ne \cos \phi \cos \lambda \\ Ne \cos \phi \sin \lambda \\ \frac{b^2}{a^2} Ne \sin \phi \end{bmatrix},$$

$$\begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} = \begin{bmatrix} \frac{x}{Ne} \\ \frac{y}{Ne} \\ \left(\frac{a}{b}\right)^2 \frac{z}{Ne} \end{bmatrix} = \begin{bmatrix} \cos\phi \cos\lambda \\ \cos\phi \sin\lambda \\ \sin\phi \end{bmatrix} .$$

From Figure 1, it follows immediately that

$$\begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} + H \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} = \begin{bmatrix} (Ne + H) \cos\phi \cos\lambda \\ (Ne + H) \cos\phi \sin\lambda \\ \left(\frac{b^2}{a^2} Ne + H\right) \sin\phi \end{bmatrix} .$$

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13. ABSTRACT *(Maximum 200 words)*

This report derives a new and improved technique for computing the geodetic latitude and altitude of an arbitrary point in space assuming the Earth is an ellipsoid of revolution. This method theoretically computes the exact values of the geodetic latitude and altitude. All the programs currently in use utilize approximation techniques for computing the quantities mentioned above. The method is simple and efficient and is strongly recommended for implementation.

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